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Least Square Based for First, Second and Third Order Modelling

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Abstract

Using experimental data from a single-phase induction motor, this paper investigates the Least Square (LS) method for modeling systems of different orders, specifically first, second, and third order. Voltage makes up the input data, and current signals derived from LS-based parameter estimation make up the output data. Examining how model-order variations affect system behavior under step and frequency response conditions is the aim of this study. The results show that while higher-order models, especially the third-order model, show overshoot and larger oscillation amplitude, the first-order model generates a more stable and quicker response. Additionally, frequency response tests show that amplitude fluctuations increase with model order, suggesting a decrease in robustness to noise. Overall, the findings support the idea that while higher-order models are better at capturing system dynamics, their accuracy and stability depend on more robust system design.
***Keywords**: Least Square Method, Model Order, Parameter Estimation, Single-Phase Induction Motor*

1. INTRODUCTION

Modeling in general can be carried out using two main approaches: by deriving models from the fundamental laws of physics and chemistry or by utilizing process data. Each of these methods has its own strengths and limitations. The physics- and chemistry-based approach offers the advantage of generating a model without the need to physically build or operate the system first. By understanding the dynamic principles governing the system, a reliable model can be developed, making the process relatively cost-effective. However, this method requires specialized expertise in mathematical modeling and regression techniques, which can make the process more complex. Furthermore, the accuracy of the final model may be impacted by ignoring specific conditions, such as environmental factors and usage duration. Process parameter estimation is used in the second modeling approach. The benefit of this method is that it creates a more modern and adaptable model that can accommodate different system orders. The disadvantage is that it costs more because it needs a physical system that is fully functional in order to produce the required process data.

The least-squares-based method can generate models using a multi-order system approach. However, the limited understanding of how different model orders affect performance reduces the potential benefits of such variations. Therefore, this study proposes to explore model-order variations using the Least-Squares-Based estimation method. The objective is to produce models of various orders and analyses their corresponding system responses. This research focuses on single-phase induction motor process data, where voltage serves as the input and current as the output.

Recent studies have highlighted the significant impact of model-order variation on the accuracy of least-squares-based system identification. Cantera-Cantera et al. (2023) compared classic LS and its orthogonal-distance variant across models of different orders, showing

distinct performance outcomes (Cantera-cantera et al. 2023). Likewise, Cortiella et al. (2020) demonstrated that regularized LS methods rely heavily on selecting the right model structure (Cortiella, Park, and Doostan 2020), while Chen et al. (2023) emphasized how order assumptions affect identifiability in multi-task LS models (Chen et al. n.d.). Ivanov and Zhdanov (2021) further noted that fractional-order systems require careful order selection for reliable estimation (Ivanov 2021). These findings reinforce the relevance of this study, which applies a least-squares-based estimation method with varying model orders to single-phase induction motor data to compare response accuracy, overfitting tendencies, and computational efficiency.

2. REVIEW OF LITERATURE

Relevant research on first, second and third order model research and the previous use of Least Sqaure Based can be seen in some of the studies below;

M. Vanithasri, R. Balamurugan and L. Lakshminarasimman: research on the use of Modified Radial Movement Optimization (MRMO) in estimating fuel cost function parameters of thermal power plants. This study uses the first, second and third order system models in the calculation of the coefficient of fuel curve function in thermal plants. (Vanithasri, Balamurugan, and Lakshminarasimman 2016)

G. Z. Kyzas, N. K. Lazaridis, and A. Ch. Mitropoulos: Research on the use of natural materials, namely Greek Coffee Waste (coffee waste) as an efficient, economical and environmentally friendly material for the process of removing dyes in the decolorization of liquid waste in the textile industry. First, second and third order models were carried out to determine the rate and mechanism of the dye absorption process. (Kyzas, Lazaridis, and Mitropoulos 2012)

Dr. Randall W. Jensen, Lawrence H. Putnam Sr, and William Roetzheim: This study compares first, second and third order models to maintain comparative consistency between cost estimates and software development schedules. (Jensen, Putnam, and Roetzheim 2006)

Yuhong Sheng, Kai Yao, and Xiaowei Chen : This research uses Least Square Estimation as an approach model to solve the principle of minimum noise. This method is able to produce stable and accurate parameter estimates even though the system contains non-probabilistic uncertainties. (Sheng, Yao, and Chen 2020)

Yuji Saito, Taku Nonomura, Keigo Yamada, Kumi Nakai, Daisuke Tsubakino, Keisuke Asai and Yasuo Sasaki: This study aims to improve the efficiency and accuracy in determining the optimal location of a limited number of sensors used to estimate system conditions (state estimation) through the Least Squares Estimation (LSE) method.(Saito et al. 2021)

Zhicheng He, Ziming Yang, Xiangyu Cui, and Eric Li**:** In this study, a new method for estimating SOC is proposed online, which combines the Adaptive Extended Kalman Filter (AEKF) and parameter identification algorithm based on Adaptive Recursive Least Squares (RLS). The test results show that the proposed method is able to accurately estimate SOC and has strong robustness to disturbances and changes in system conditions. (He et al. 2020)

3. METHOD

The Least-Square Based method uses the basic linear equation of discrete shapes in estimating parameters to get a model, namely with the following equation (1).

$$y(k) = -a_1 y(k-1) - a_2 y(k-2) - \cdots - a_m y(k-m) + b_1 u(k-1) + b_2 u(k-2) + \cdots + b_n u(k-n) \quad (1)$$

where:

y(*k*) : system output (e.g. motor speed or current) at the *k time*

u(*k*) : system input (e.g. voltage) at the k time

a_1, a_2, b_1, b_2 : The parameters of the model to be estimated

e(*k*) : noise or interference

The equivalence of equation (1) above with the form of matrix notation can be seen in Equation (2) below. This equation notation (5) is written with each Y as an output vector, φ as a matrix of input and output before, and θ as an estimation parameter.

$$Y = \varphi\theta + E \quad (2)$$

With the definition of each notation, namely:

$$Y = \begin{bmatrix} y(3) \\ y(4) \\ \vdots \\ y(N) \end{bmatrix} \quad (3)$$

$$\varphi = \begin{bmatrix} -y(2) & -y(1) & u(2) & u(1) \\ -y(3) & -y(2) & u(3) & u(2) \\ \vdots & \vdots & \vdots & \vdots \\ -y(N-1) & -y(N) & u(N-1) & u(N-2) \end{bmatrix} \quad (4)$$

$$\theta = \begin{bmatrix} a_1 \\ a_2 \\ b_1 \\ b_2 \end{bmatrix} \quad (5)$$

To obtain the estimation parameters, the equation must first be manipulated based on the matrix form derived from Equation (2). The result of the manipulation of the equation is obtained Equation (6) with θ as the definition of the parameter estimation results.

$$\hat{\theta} = (\varphi^T \varphi)^{-1} \varphi^T Y \quad (6)$$

The result of equation (6) above is an equation to produce an estimation parameter using Least-Square Based.

To produce parameter estimation with various order approaches, the parameter estimation process will use various discrete forms of Equation (1). To produce an estimate of the first-order parameter, it can be done using equation (7) below.

$$y(k) = -a_1 y(k-1) + b_1 u(k-1) \quad (7)$$

To generate an estimate of second-order parameters, it can be done using equation (8) below.

$$y(k) = -a_1 y(k-1) - a_2 y(k-2) + b_1 u(k-1) + b_2 u(k-2) \quad (8)$$

To obtain an estimate of third-order parameters, it can be done using equation (9) below.

$$y(k) = -a_1 y(k-1) - a_2 y(k-2) - a_3 y(k-3) + b_1 u(k-1) + b_2 u(k-2) + b_3 u(k-3) \quad (9)$$

From the three equations above, Equations (7), (8) and (9), it can be seen that the variation in parameters is obtained from the variation of data collection for the estimation process. The higher the order, the more data is used.

For validation of the accuracy of the model estimation results, the results are carried out using the following Equation (10).

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(yi - yp)^2 \quad (10)$$

This validation is using Means Square Error (MSE) where yi is the real system output of the estimated process data and yp is the prediction system output of the estimated results. From Equation (10), this becomes the basis for whether the resulting model is close to accurate or not.

4. RESULT AND DISCUSSION

First Order Model

From the modeling process using Least Square Estimation, a discrete transfer function was obtained with a first-order model approach using a time sampling of 0.1 as shown in Equation (11) below.

$$G(z) = \frac{0.003271}{z - 0.8014} \quad (11)$$

The test between the real output data and the model output data can be seen in Figure 1 below.

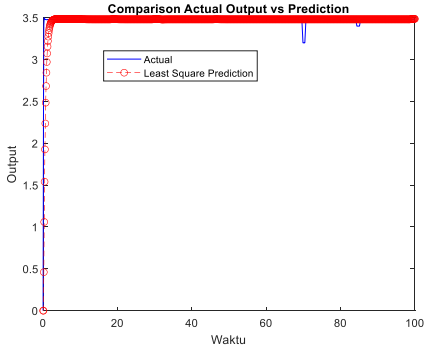


Fig 1. Comparison of LSE model output with actual

From figure 1 above, the output of the least square prediction model has followed the actual output. The amount of error level validation can be seen in equation (12) about the following MSE.

$$MSE = 0.0307 \quad (12)$$

From the MSE quantity, it is obtained that the model results have a relatively small average error so that it can be concluded that the model results of the approach using Least Square Estimation for order one produces an accurate model. The prediction error can be seen in Figure 2 below.

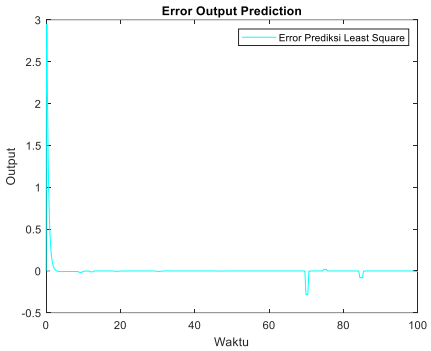


Fig 2. Prediction Error is close to Zero.

From Figure 2, it can be seen that the largest error is seen in the 2nd second. However, this error is quite large with a maximum value of 3.

To perform Simulink simulations, the model must be a continuous transfer function. Therefore, the process of conversion from the discrete function of Equation (11) above to a continuous transfer function can be seen in the following equation (13).

$$G(s) = \frac{0.3646}{s+22.14} \tag{13}$$

Second Order Model

From the modeling process using Least Square Estimation, a discrete transfer function was obtained with a second-order model approach using a time sampling of 0.1 as shown in Equation (14) below.

$$G(z) = \frac{0.00306\,z-0.002353}{z^2-1.239\,z+0.2819} \tag{14}$$

The test between the real output data and the model output data can be seen in Figure 3 below.

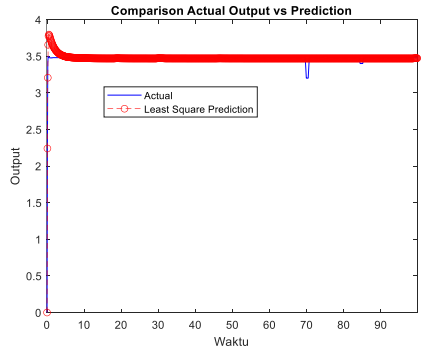


Fig 3. Comparison of LSE model output with actual

From figure 3 above, the output of the least square prediction model follows the actual output. The amount of error level validation can be seen in equation (15) about the following MSE.

$$MSE = 0.001714 \tag{15}$$

From the MSE quantity, it is obtained that the model results have a relatively small average error so that it can be concluded that the model results of the approach using Least Square

Estimation for order two produces an accurate model. The prediction error can be seen in Figure 4 below.

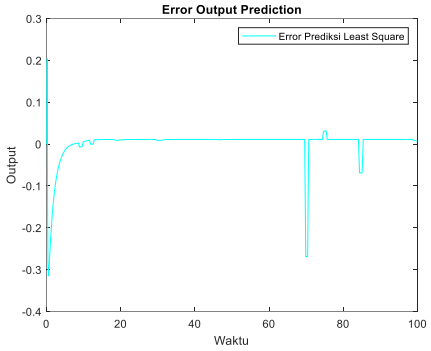


Fig 4. Prediction Error Approaches Zero.

From figure 4, the largest error can be seen at the 2nd and 70th seconds. However, this error is relatively small with a maximum value of 0.3.

To perform simulink simulations, the model must be a continuous transfer function. Therefore, the process of conversion from the discrete function of Equation (14) above to a continuous transfer function can be seen in the following equation (16).

$$G(s) = \frac{0.04702\ s+0.1254}{s^2+12.66\ s+7.637} \quad (16)$$

Third Order Model

From the modeling process using Least Square Estimation, a discrete transfer function was obtained with a third-order model approach using a time sampling of 0.1 as shown in Equation (17) below.

$$G(z) = \frac{-0.007982\ z^2+\ 0.009326\ z-0.0001365}{z^3-1.673\ z^2+0.8176\ z+0.07133} \quad (17)$$

The test between the real output data and the model output data can be seen in Figure 5 below.

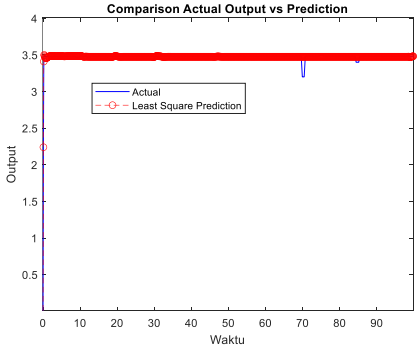


Fig 5. Comparison of LSE model output with actual

From figure 5 above, the output of the least square prediction model follows the actual output. The magnitude of the error level validation can be seen in equation (18) about the following MSE.

$$MSE = 0.000616 \quad (18)$$

From the magnitude of MSE, it is obtained that the model results have a relatively small average error so that it can be concluded that the model results of the approach using Least

Square Estimation for order three produces an accurate model. The prediction error can be seen in Figure 6 below.

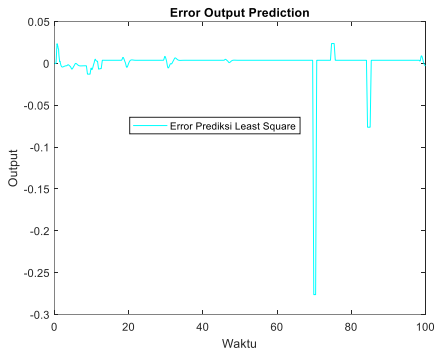


Fig 6. Prediction Error is close to Zero.

From figure 6, it can be seen that the largest error is seen at the 70th second. However, this error is relatively small with a maximum value of 0.26.

To perform Simulink simulations, the model must be a continuous transfer function. Therefore, the conversion process from the discrete function of Equation (17) above to a continuous transfer function can be seen in the following equation (19).

$$G(s) = \frac{-0.08837s^2 - 2.514\,s + 3.725}{s^3 + 26.4\,s^2 + 106.9\;s + 226.4} \tag{19}$$

Table 1. MSE and Transfer Function

No.	Model	Transfer Function	MSE
1.	First Order	$G(s) = \frac{0.3646}{s + 22.14}$	0.0307
2.	Second Order	$G(s) = \frac{0.04702\,s + 0.1254}{s^2 + 12.66\,s + 7.637}$	0.001714
3.	Third Order	$G(s) = \frac{-0.08837s^2 - 2.514\,s + 3.725}{s^3 + 26.4\,s^2 + 106.9\;s + 226.4}$	0.000616

From the first, second and third order transfer functions obtained from table 1, experimental tests were carried out on the transfer functions. The experiment was carried out using two test signals, namely the step test signal and the sine test signal. This aims to see the response of the three models of the order approach.

The first test was carried out using a step signal with an input parameter of 220 Volts and a duration of 10 seconds. The results of the test can be seen in Figure 7 below.

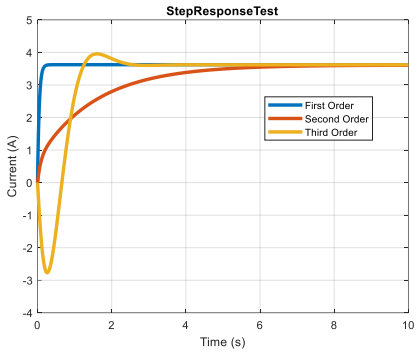


Fig 7. Step Response Testing

From the test results, it was obtained that the response of the first-order model had higher stability compared to the response of the second- and third-order models. In addition, the response of the first-order model is able to reach a steady state faster compared to both. In the third-order response, it was seen that there was an overshoot so it was necessary to prepare a system that was more robust and had a fairly wide range of capabilities. This means that the system modeled with a third-order approach must have a wider range of resistance to interference characteristics compared to first-order and second-order systems.

The second test is by using sinusoidal frequency signal testing with amplitude parameters of 220 Volts, frequency of 1 Hertz and duration of 1 second. The test results can be seen in Figure 8 below.

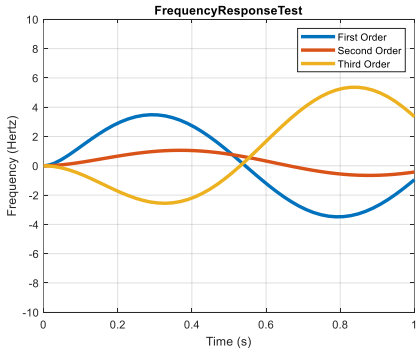


Fig 8. Frequency Response Testing

From the test results, the third-order response has a greater amplitude compared to the first-order and second-order responses. The amount of amplitude produced makes the system oscillate higher. Tis is a characteristic of this third-order system. From this test, it was obtained that the larger the order of the model approach, the greater the oscillation amplitude.

5. CONCLUSION

To estimate the second-order parameters, the results of the experiment show that the Least Square Modeling method can effectively generate models with various system orders, including first-order, second-order, and third-order models. Based on the transfer function experiments, the step response analysis indicates that the lower the model order, the more stable the system response becomes. Conversely, higher-order models tend to produce overshoot in the response. In the frequency response test, it was observed that increasing the model order leads to a higher frequency amplitude. Therefore, systems modeled with higher-order approaches must be designed with greater robustness to handle noise effectively.

6. ACKNOWLEDGEMENTS

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